FLUTTUAZIONI IN SISTEMI CON DEGENERAZIONE CONTINUA

$\varphi_\alpha \qquad \alpha = 1, \dots, n$

in He^4 e superconduttori $\quad \psi = \varphi_1 + i\varphi_2 \quad$ FUNZIONE D'ONDA DEL CONDENSATO

$\psi \to e^{i\omega}\psi$

Anche $\vec{m} = (m_x, m_y)$ MAGNETE CON PIANO FACILE

$\langle\varphi\rangle \neq 0 \;\; T < T_c \quad$ e $\quad \langle\varphi\rangle = 0$ per $T > T_c \qquad$ ROTTURA DI SIMMETRIA

$\longrightarrow$ Eccitazioni di Goldstone

$\Phi = \frac{1}{V}\int dx \left[f(\varphi^2) - \underline{h}\cdot\underline{\varphi} \right] \qquad$ POT. TERMODINAMICO

$2\underline{\varphi}\, f'(\varphi^2) = \underline{h} \qquad$ CONDIZ. DI ESTREMO

SUSCETTIVITA':

$$\chi_{\alpha\beta} = \frac{\partial \varphi_\alpha}{\partial h_\beta} = \chi_{\parallel}(h^2)\, n_\alpha n_\beta + \chi_\perp(h^2)\left(\delta_{\alpha\beta} - n_\alpha n_\beta\right)$$

con $\underline{n} = \underline{h}/|\underline{h}|$ $\qquad$ SUSC. LONGITUDINALE $\qquad$ SUSC. TRASVERSALE

per $\begin{pmatrix} T > T_c \\ h = 0 \end{pmatrix} \quad \chi_\parallel^{(0)} = \chi_\perp^{(0)} \longrightarrow \quad \chi_{\alpha\beta}^{(0)} \propto \delta_{\alpha\beta}$

per $T < T_c$ e $h \to 0 \quad \underline{\varphi} = \underline{\varphi}_s$ ricorda la direzione di $\underline{h}$

e $\boxed{f'(\varphi_s^2) = 0}$

$$\chi^{-1}_{\alpha\beta} = \frac{\partial h_\alpha}{\partial \varphi_\beta} = \frac{1}{\chi_\parallel} n_\alpha n_\beta + \frac{1}{\chi_\perp}\left(\delta_{\alpha\beta} - n_\alpha n_\beta\right)$$

$\chi_\perp^{-1} = 2f'(\varphi^2) \qquad \chi_\parallel^{-1} = 4\varphi^2 f''(\varphi^2) + 2f'(\varphi^2)$

$\downarrow$

$\chi_\perp = \varphi/h$

rotazione

• Per $T<T_c$ e $h\to 0$ $\chi_\perp \to \infty$ INSTABILITÀ RISPETTO ALLE FLUTTUAZ. TRASVERSALI

$$\delta\underline{\Phi} = \frac{1}{2}\cancel{\int dx}\; \chi^{-1}_{\alpha\beta}\,\delta\varphi_\alpha\,\delta\varphi_\beta = \frac{1}{2}\left(\frac{1}{\chi_\perp}\delta\varphi_\perp^2 + \frac{1}{\chi_\parallel}\delta\varphi_\parallel^2\right) \quad !$$

$$\chi^{-1}_{\alpha\beta} = \frac{\partial h_\alpha}{\partial\varphi_\beta} = \frac{\partial^2\underline{\Phi}}{\partial\varphi_\alpha\partial\varphi_\beta}$$

$\Big\{$ divergenza di $\chi_\perp$ $\equiv$ equilibrio indifferente rispetto alle rotazioni di $\underline{\varphi}$ con $|\underline{\varphi}| = \varphi_s$!

Sviluppiamo fino all'ordine $\delta\varphi_\perp^4$

$$|\underline{\varphi}_s+\delta\underline{\varphi}|^2 = |\underline{\varphi}_s|^2 + 2\underline{\varphi}_s\cdot\delta\underline{\varphi} + \delta\underline{\varphi}\cdot\delta\underline{\varphi}$$

$$|\varphi_s+\delta\underline{\varphi}|^2 - |\varphi_s|^2 \simeq 2\varphi_s\delta\varphi_\parallel + \delta\varphi_\perp^2 + \delta\varphi_\parallel^2$$

$$\delta\underline{\Phi} = \frac{1}{2\chi_\perp}\delta\varphi_\perp^2 + \frac{1}{2\chi_\parallel}\frac{\boxed{\delta(\varphi^2)}^2}{4\varphi_s^2}$$

$\delta(\varphi^2) \simeq 2\varphi_s\delta\varphi_\parallel + \delta\varphi_\perp^2$ (con $f'(\varphi_s^2)\varphi_s \to 0$)

per $\delta\varphi_\perp$ fissato il minimo $\delta\underline{\Phi}$ si ha per $\delta(\varphi^2)=0$

PRINCIPIO DI ~~FLUTTUAZIONE DI~~ CONSERVAZIONE DEL MODULO

FLUTTUAZIONI TRASVERSALI

$$\delta\underline{\Phi}_\perp = \frac{1}{2V}\int\left[\frac{1}{\chi_\perp}\delta\varphi_\perp^2 + c\,(\nabla\delta\varphi_\perp)^2\right]d\underline{x}$$

RIGIDITÀ RISPETTO ALLE ROTAZIONI NON OMOGENEE DI $\underline{\varphi}$

Nel superfluido $\Psi = |\Psi|e^{iw} \simeq |\Psi|(1-iw)$

$$\vec{v}_s = \frac{\hbar}{m}\nabla w \simeq \frac{\hbar}{m|\Psi|}\nabla\delta\varphi_\perp$$

$$\frac{1}{2}c(\nabla\delta\varphi_\perp)^2 = \frac{1}{2}\rho_s v_s^2 \quad \text{EN. CINETICA}$$

$$c = \rho_s\frac{\hbar^2}{m^2|\Psi|^2}$$

SISTEMI DEGENERI BIDIMENSIONALI

$$\delta\Phi_\perp = \frac{1}{2V}\int \left[\frac{h}{\varphi}\delta\varphi_\perp^2 + J(\nabla\delta\varphi_\perp)^2\right] d^d\underline{x}$$

$$\rightarrow \langle|\delta\varphi_\perp(q)|^2\rangle = \frac{T}{\frac{h}{\varphi} + Jq^2}$$

$$G_\perp(\underline{x},\underline{x}') = \langle\delta\varphi_\perp(\underline{x})\delta\varphi_\perp(\underline{x}')\rangle = \frac{1}{(2\pi)^d}\int\langle|\delta\varphi_\perp(q)|^2\rangle e^{i\underline{q}\cdot(\underline{x}-\underline{x}')} d^d\underline{q}$$

$$\rightarrow \langle\delta\varphi_\perp^2(\underline{x})\rangle = G_\perp(\underline{x},\underline{x}) = \frac{T}{(2\pi)^d}\int\frac{d^dq}{\frac{h}{\varphi}+Jq^2}$$

$$\approx \frac{T\Omega_d}{(2\pi)^d}\int\frac{q^{d-1}dq}{\frac{h}{\varphi}+Jq^2}$$

se per $h\to 0$, $\varphi\to\varphi_0$: $\quad G_\perp(\underline{x},\underline{x}) \sim \int_0 q^{d-3}dq$

DIVERGE PER $d-3 \leq -1$

$$\boxed{d\leq 2}$$

$$\rightarrow \quad \varphi_0 = \langle\varphi\rangle \xrightarrow[h\to 0]{} 0 \quad \text{in } d\leq 2$$

NO PARAMETRO D'ORDINE

$$\underline{\varphi} = \varphi_{MAX}\,\underline{n} \qquad (\varphi_{MAX}=1)$$

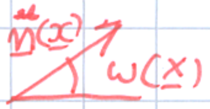
$$\Phi = \Phi_0 + \int\left(\frac{1}{2}J(\nabla\underline{n})^2 - \underline{h}\cdot\underline{n}\right) d^d\underline{x}$$

$$P[\underline{n}(\underline{x})] = \frac{1}{Z}e^{-\Phi[\underline{n}(\underline{x})]/T} \quad !$$

$h=0 \quad \langle\underline{n}\rangle = 0 \qquad G_{\alpha\beta}(\underline{x},\underline{x}') = \langle n_\alpha(x)n_\beta(x')\rangle =$

$$= \delta_{\alpha\beta}G(\underline{x},\underline{x}') = \frac{\delta_{\alpha\beta}}{n}\langle\underline{n}(x)\cdot\underline{n}(x')\rangle$$

$n=2$



$$2G(\underline{x},\underline{x}') = \langle \cos(\omega(\underline{x}) - \omega(\underline{x}')) \rangle$$

$$= \langle e^{i(\omega(\underline{x}) - \omega(\underline{x}'))} \rangle$$

$$\Phi = \Phi_0 + \frac{J}{2} \int [\nabla \omega(\underline{x})]^2 d\underline{x}$$

$$\rightarrow \boxed{P[\omega(\underline{x})] = \frac{1}{Z^{-1}} e^{-\frac{J}{2T} \int [\nabla \omega(\underline{x})]^2 d\underline{x}}}$$

GAUSS !!

$$2G(\underline{x},\underline{x}') = \langle e^{i(\omega(\underline{x}) - \omega(\underline{x}'))} \rangle =$$

$$e^{-\frac{1}{2}\langle [\omega(\underline{x}) - \omega(\underline{x}')]^2 \rangle}$$

Ma $\langle [\omega(\underline{x}) - \omega(\underline{x}')]^2 \rangle = 2[\langle \omega^2(\underline{x}) \rangle - \langle \omega(\underline{x}) \omega(\underline{x}') \rangle]$

In trasformata di Fourier

$$\Phi = \Phi_0 + \frac{J}{2} \int q^2 |\omega_{\underline{q}}|^2 \frac{d^dq}{(2\pi)^d}$$

$$\langle |\omega_{\underline{q}}|^2 \rangle = \frac{T}{\pi J q^2} \qquad d=2$$

$$= \frac{2T}{Jq^2} \qquad d=1$$

$$\langle [\omega(\underline{x}) - \omega(\underline{x}')]^2 \rangle = \frac{2}{(2\pi)^d} \int d^d\underline{q} \, \langle |\omega_{\underline{q}}|^2 \rangle \left[1 - e^{i\underline{q}\cdot(\underline{x} - \underline{x}')}\right]$$

(0)

$d=2$

$$(\bullet) = \frac{T}{\pi J}\int_0^{\text{"}\infty\text{"}} \frac{dq}{q}\left[1 - J_0(q|\underline{x}-\underline{x}'|)\right] \overset{\infty \Rightarrow \frac{1}{a}}{\approx} \frac{T}{\pi J}\ln\frac{|\underline{x}-\underline{x}'|}{a}$$

FUNZ. DI BESSEL

$d=1$

$$(\bullet) = \frac{2T}{\pi J}\int_0^{\infty} \frac{dq}{q^2}\left[1-\cos(q|\underline{x}-\underline{x}'|)\right] = \frac{2T}{J}|\underline{x}-\underline{x}'|$$

$d=2$ $\quad 2G(\underline{x},\underline{x}') \propto \left[\frac{|\underline{x}-\underline{x}'|}{a}\right]^{-T/2\pi J}$ $\quad$ LEGGE A POTENZA $\quad \xi = \infty$

$\boxed{d=1}$ $\quad 2G(\underline{x},\underline{x}') \propto e^{-\frac{T}{2Ja}|\underline{x}-\underline{x}'|}$ $\quad \boxed{\xi = \frac{2Ja}{T}}$

In $d=2$ $\longrightarrow$ $\boxed{\text{RIGIDITÀ}}$ a bassa T

se il raggio delle interazioni è a ad alta T:

$$\langle \underline{\varphi}(\underline{x})\underline{\varphi}(\underline{x}+\underline{a})\rangle \sim \frac{J}{T} \ll 1 \qquad \text{per } T \gg J$$

$$\langle \underline{\varphi}(\underline{x})\underline{\varphi}(\underline{x}')\rangle \sim \langle \underline{\varphi}(x)\underline{\varphi}(\underline{x}+\underline{a})\rangle\langle \underline{\varphi}(\underline{x}+\underline{a})\underline{\varphi}(\underline{x}+2\underline{a})\rangle$$
$$\cdots\langle \underline{\varphi}(\underline{x}+(N-1)\underline{a})\underline{\varphi}(\underline{x}')\rangle \sim \left(\frac{J}{T}\right)^{|\underline{x}-\underline{x}'|/a}$$
$$\approx e^{-\frac{|\underline{x}-\underline{x}'|}{a}\ln(T/J)}$$

$$\xi \approx \frac{a}{\ln T/J}$$

$d=2$

$d=1$

TRANSIZIONE PER $d=2$!!!

$$DW = \frac{m}{\hbar}\underline{v}_s \qquad \boxed{J = \frac{\hbar^2}{m^2}\rho_s}$$

$$\Phi = \Phi_0 + \int \frac{\rho_s \underline{v}_s^2}{2}\, d^d\underline{x} \qquad \underline{j}_s = \rho_s \underline{v}_s$$

VORTICI NEI SISTEMI A 2 COMPONENTI

$-\pi \leq \omega \leq \pi$

$\underline{A}_{\underline{a}}(\underline{x}) = \omega(\underline{x}+\underline{a}) - \omega(\underline{x}) \qquad -\pi \leq \underline{A}_{\underline{a}}(\underline{x}) \leq \pi$

$\omega_L(\underline{x}) = \omega_{\underline{0}} + \sum_{(\underline{x}', \underline{x}'+\underline{a}) \in L} \underline{A}_{\underline{a}}(\underline{x}')$ L CAMMINO da $\underline{0}$ a $\underline{x}$

$\omega_L(\underline{x}) - \omega_{L'}(\underline{x}) = 2\pi m \longrightarrow \sum_{\Gamma} \underline{A}_{\underline{a}}(\underline{x}) = 2\pi m$

Ma $\sum_{\Gamma} = \sum_{\Gamma_1} + \sum_{\Gamma_2} + \ldots + \sum_{\Gamma_n} = 2\pi \sum_{\underline{\tilde{x}}} Q(\underline{\tilde{x}})$

intero

VORTICI DISCRETI DEL CAMPO $\underline{A}_{\underline{a}}(\underline{x})$

$\underline{A}_{\underline{a}}(\underline{x})$ e $\omega(\underline{x})$ SONO DETERMINATI A MENO DI UNA TRASF. DI GAUGE

$\underline{A}'_{\underline{a}}(\underline{x}) = \underline{A}_{\underline{a}}(\underline{x}) + \eta(\underline{x}+\underline{a}) - \eta(\underline{x})$

$\omega'(\underline{x}) = \omega(\underline{x}) + \eta(\underline{x})$

Introduciamo il potenziale magnetico $\psi(\underline{\tilde{x}})$:

$\Delta\psi(\underline{\tilde{x}}) = 2\pi Q(\underline{\tilde{x}})$ con $\Delta\psi(\underline{\tilde{x}}) = \sum_{\underline{\tilde{a}}} \left[\psi(\underline{\tilde{x}}+\underline{\tilde{a}}) - \psi(\underline{\tilde{x}})\right]$

LAPLACIANO DISCRETO

$\underline{\tilde{A}}_{\underline{\tilde{a}}}(\underline{\tilde{x}}) \equiv \psi(\underline{\tilde{x}}+\underline{\tilde{a}}) - \psi(\underline{\tilde{x}}) = \underline{A}_{\underline{a}}(\underline{x})$

$\psi(\underline{\tilde{x}}) = 2\pi \sum_{\underline{\tilde{x}}'} D(\underline{\tilde{x}}, \underline{\tilde{x}}') Q(\underline{\tilde{x}}') \qquad D(\underline{\tilde{x}}, \underline{\tilde{x}}') \simeq \frac{1}{2\pi} \ln|\underline{\tilde{x}} - \underline{\tilde{x}}'|$

• HAMILTONIANA

$$H=\sum_{\underline{x},\underline{a}} f(\underline{A}_{\underline{a}}(\underline{x}))$$ con f periodica di periodo 2π

p. es. $H=-J\sum_{\underline{x},\underline{a}} \cos \underline{A}_{\underline{a}}(\underline{x})$

per $\underline{A}=0$, $f(\underline{A})$ ha un minimo assoluto

STATO FONDAM. $\underline{A}_{\underline{a}}(\underline{x})=0 \to \omega(\underline{x})$ è indip. da $\underline{x} \to Q(\tilde{x})=0$

TRASF. DI GAUGE

$\underline{A}_{\underline{a}}(\underline{x})=\eta(\underline{x}+\underline{a})-\eta(\underline{x})$

STATI DEBOLM. ECCITATI

sia $Q(\tilde{x})=Q_0$ per $\tilde{x}=\tilde{x}_0$ per $\tilde{x}\gg\tilde{x}_0$ $\underline{A}_{\underline{a}}(\underline{x})\approx\frac{Q_0}{|\tilde{x}-\tilde{x}_0|}$

$$f(\underline{A})\approx f(0)+\frac{J}{2}\underline{A}^2 \longrightarrow \delta E\approx\frac{JQ_0^2}{2}\sum_{\underline{x}}\frac{1}{|\underline{x}-\underline{x}_0|^2}$$

DIVERGE LOG

CON n VORTICI

$$\psi(\tilde{x})\simeq\sum Q(\tilde{x}_i)\ln|\tilde{x}-\tilde{x}_i| \overset{|\tilde{x}|\gg|\tilde{x}_i|}{\approx} \ln|\tilde{x}|\sum Q(\tilde{x}_i)$$

se $\sum Q(\tilde{x}_i)=0$ SCOMPARE IL CONTRIB. LOGARITMICO.

OGGETTO PIÙ SEMPLICE: MOLECOLA DI DUE VORTICI

$$Q(\tilde{x}_1)=-Q(\tilde{x}_2)=Q$$

$$\psi(\tilde{x})\approx Q\ln\frac{|\tilde{x}-\tilde{x}_1|}{|\tilde{x}-\tilde{x}_2|}$$

$$\delta E(\tilde{x}_1-\tilde{x}_2)\approx 2\pi JQ^2\ln\frac{|\tilde{x}_1-\tilde{x}_2|}{a}$$

MOLECOLE DI VA $\searrow$ A BASSA T

DISSOCIAZIONE (BKT)
||
TRANSIZIONE DI FASE

$$\delta\Phi = \delta E - T\delta S \qquad \begin{cases} \delta E = \pi J \ln \frac{R}{a} \\ \delta S = 2 \ln \frac{R}{a} \end{cases}$$

(LOG. DELL'AREA)

$$\delta\Phi = (\pi J - 2T) \ln \frac{R}{a}$$

per $T > \frac{\pi}{2} J = T_{BKT}$ UN VORTICE ISOLATO È <u>VANTAGGIOSO</u>

$$\rho_s = \frac{J m^2}{\hbar^2}$$ per He^4