feature article

# The *vis viva* dispute: A controversy at the dawn of dynamics

George E. Smith

**The need to augment Newtonian mechanics to encompass systems more complex than collections of point masses engendered a century-long dispute about conservation principles.**

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**Mechanics as a science of motion,** as distinguished from a science of machines such as the lever and windlass, started early in the 17th century. By the middle of the next century it had become clear that Isaac Newton's three laws suffice for the motions of "point masses," but it was not yet clear how—and indeed whether—those laws could be extended to handle the motions of fluids or rigid bodies. Thus the 18th century saw new laws such as the principle of least action proposed and disputed. The most celebrated of those disputes, concerning the conservation of *vis viva* (Latin for "living force" and akin to what we now call kinetic energy), was already under way by 1686, the year before Newton published his laws of motion in the *Principia*.[1]

The *vis viva* controversy started as a dispute between Gottfried Wilhelm Leibniz (1646–1716) and followers of René Descartes (1596–1650). It continued throughout the 18th century, becoming the topic of several prize competitions.[2] In 1788, long after the initial partisans had passed from the scene, Joseph Louis Lagrange (1736–1813) opened part II of his *Mécanique analytique* by raising the *vis viva* question once again.[3]

The controversy is now usually portrayed as a dispute about the conservation of $mv$ (or momentum) versus the conservation of $mv^2$ (or kinetic energy). In fact, it was not that simple—which helps explain why it continued for so long. The best way to appreciate the different issues is by reviewing how the controversy got started. First, however, we must remove some anachronisms implicit in viewing the argument as $mv$ versus $mv^2$.

Newtonian mass did not become part of the controversy until well into the 18th century. To ignore that is to lose sight of the novelty of Newton's concept. Newton first introduced mass (Latin *massa*) in the *Principia* as short for "quantity of matter." Initially he had considered "heaviness" (Latin *pondus*). In introducing mass he emphasized that "very accurate experiments with pendulums" had shown that it is proportional to weight.

The standard term before Newton was "bulk" (Latin *moles*). He himself retained that older term in his only published solution for the motion of colliding spheres, in his *Arithmetica universalis*,[4] which appeared in Latin in 1707. "Bulk" reflected the widespread view, held by both Leibniz and the Cartesians, that gravity and weight involve ethereal matter pressing down on solid matter in such a way that weight is proportional to the quantity of solid matter.

The other anachronism in $mv$ and $mv^2$ is the use of symbols at all. Until the calculus took over during the 18th century, quantities were represented not by algebraic symbols but by geometric constructs like lines and areas, and relationships among quantities were expressed not as equations but as proportions. The two quantities originally entering into the *vis viva* dispute were "motion," taken to be the product of bulk and velocity or speed, and, following Leibniz, *vis viva*, the product of bulk and speed squared.

## Galileo's *Discorsi*

The notion that the square of speed is important derives from three tenets central to the account of "local motion" given by Galileo Galilei (1564–1642) in his *Dialogues Concerning Two New Sciences,* which appeared in 1638.[5]

1. In the absence of resisting media, vertical fall is a uniformly accelerated motion, and hence the square of the speed acquired during fall is proportional to the height of fall.
2. In the absence of resisting media, the speed acquired during fall from rest is precisely sufficient to raise an object back to its original height, but no higher.
3. The speed acquired in fall along an inclined plane from a given height is the same regardless of the inclination of the plane.

The last of these three tenets, which I will anachronistically call Galileo's principle of path-independence, contributes crucially to the concept of *vis viva* by giving the square of the speed a generality that it would otherwise lack. He originally introduced this principle as an assumption. The posthumous edition of the *Discorsi* offered a defense based on the magnitude of the vertically suspended weight required to hold a weight in equilibrium on an inclined plane.

Far from happy with that defense, Galileo's protegé Evangelista Torricelli (1608–47) offered a *reductio ad absurdum* derivation of it in 1644 from a principle that came to bear his name: Two weights joined together cannot begin to move by themselves unless their common center of gravity descends.[6] Three decades later, Christiaan Huygens (1629–95) stressed the importance of path-independence in his *Horologium*

**Gottfried Wilhelm Leibniz (1646–1716).**

*oscillatorium*,[7] offering a related *reductio* derivation for the more general case of curvilinear paths of descent.

## Descartes' conservation of motion

The Cartesian principle against which Leibniz offered the conservation of *vis viva* looks crazy to us at first glance. It asserts that the total quantity of motion—that is, the total quantity of bulk times speed—always remains the same. Speed here is taken to be independent of direction, not a vector quantity. Hence Descartes' principle is by no means a forerunner of our modern principle of momentum conservation. So we need to understand what he had in mind. The answer lies in Descartes' *a priori* insistence that a vacuum is impossible. All space is thus filled with matter, and the motion of any part of matter requires that the matter ahead of it be pushed forward.

Therefore, Descartes concludes, "in all movement a complete circuit of bodies moves simultaneously."[8] In *The Principles of Descartes' Philosophy,*[9] Benedict (Baruch) de Spinoza (1632–77) offers the diagram shown in figure 1 to illustrate both that conclusion and Descartes' principle of conservation of motion. The principle resembles the modern notion of continuity for incompressible fluids in that what remains constant everywhere around the circuit in the figure is the product of speed and cross-sectional area. Descartes was of course wrong; but this principle, which he considered even more fundamental than his "laws of nature," was not crazy.

The problem arose with Descartes' understanding of the mechanism that conserves motion during local changes. His first two laws of nature together asserted that motion, if not impeded, continues uniformly in a straight line. He was, in fact, the first to insist that the curvilinear motion of planets requires something to divert them from straight paths. As such, he has stronger claims than anyone else to what came to be known as the principle of inertia.

Descartes' third law of nature concerns local changes of motion:

> When a body meets another, if it has less force to continue to move in a straight line than the other has to resist it, it is turned aside in another direction, retaining its quantity of motion and changing only the direction of that motion. If, however, it has more force, it moves the other body with it, and loses as much of its motion as it gives to the other.[8]

The notion of force thus enters through interchanges of motion dictated by a contest of forces: the force to resist change of motion and the force to produce it. The latter, Descartes asserted, depends on the size of the body and its speed.

In the 1644 Latin edition of Descartes' *Principia,* he ended his discussion of interchange of motion by remarking that what happens in individual cases can be determined by calculating "how much force to move or to resist movement there is in each body, and to accept as a certainty that the one which is stronger will always produce its effect." But in the French translation three years later, he added seven supplementary rules for explicitly predicting the outcome when two "perfectly solid" bodies, perfectly separated from all others, come into contact. The third supplementary rule, for example, says that if the two bodies are of the same size, but one is moving slightly faster, then it wins the contest, transferring to the other the minimum amount of speed that ends the contest.

What is historically important about these supplementary rules is their conflict with everyday experience. Descartes recognized that conflict and offered the following defense:

> Indeed, experience often seems to contradict the rules I have just explained. However, because there cannot be any bodies in the world that are thus separated from all others, and because we seldom encounter bodies that are perfectly solid, it is very difficult to perform the calculation to determine to what extent the movement of each body may be changed by collision with others.

This defense may have been sufficient for Descartes' followers, but it challenged others to find rules of impact that experience does not contradict.

## Huygens gets it right

Among those challengers was Huygens, the son of a notable Dutch political figure whose home Descartes had often visited. During the 1650s, while still in his twenties, Huygens derived correct rules for the direct impact of hard spheres. He elected not to publish them at the time, but during a visit to London in 1661 he did describe them to various individuals who were then taking steps toward forming the Royal Society.

Near the end of 1668, two Englishmen prominent in natural philosophy and mathematics, John Wallis (1616–1703) and Christopher Wren (1632– 1723), submitted papers to the Royal Society presenting rules of impact.[10] Wallis addressed the problem of inelastic collision, and Wren considered perfectly elastic collision. At that time Henry Oldenburg, secretary of the Royal Society, solicited a paper on the topic from Huygens, which arrived in early 1669. It gave the same results as Wren's. The Wallis and Wren papers were published in the 11 January 1669 issue of *Philosophical Transactions of the Royal Society*, without mention of Huygens.

Oldenburg had reasons for soliciting Huygens's paper beyond what Huygens had said during his 1661 visit to London. By the end of the 1660s, the 39-year-old Huygens was the world's foremost figure in physics. In the 1650s, he had produced superior telescopes that had allowed him to discover Titan, the largest satellite of Saturn. That discovery was described in his 1659 book *Systema Saturnium* along with his realization that Saturn's strange protuberance, observed for decades by earlier telescopes, is in fact a ring. By the end of the 1650s he had also established the isochronism of the cycloidal pendulum, designed cycloidal pendulum clocks of great benefit to observatories, and used pendulums to measure the strength of surface gravity to four significant figures.

Huygens was the first foreigner elected as a fellow of the Royal Society, and, though a Dutch protestant, he was an academician of France's Académie Royale des Sciences from its inception in 1666. Holland has had the misfortune of producing two of the great stars in the history of physics, Huygens and Hendrik Antoon Lorentz, only to see each eclipsed by a supernova in his own time: Newton and Albert Einstein.

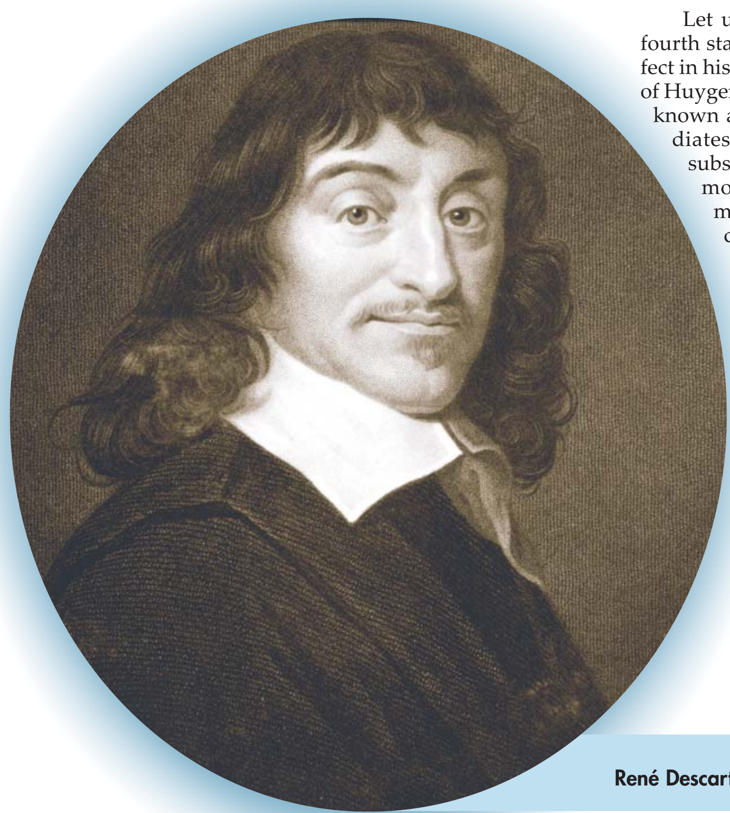
When Huygens's paper was not included in *Philosophical Transactions*, he published a condensed version in the 8 March 1669 issue of *Journal des Sçavans*. Recognizing the slight, Oldenburg quickly published a Latin translation, together with an explanation of what had transpired, in *Philosophical Transactions*.[11] This one-and-a-half-page paper solved the problem of the head-on impact of hard spheres. It ends with four consequences of that solution:

1. The quantity of motion that two hard bodies have may be increased or diminished by their collision, but when the quantity of motion in the opposite direction has been subtracted there remains always the same quantity of motion in the same direction.
2. The sum of the products obtained by multiplying the magnitude of each hard body by the square of its velocity is always the same before and after collision.
3. A hard body at rest will receive more motion from another, larger or smaller body if a third intermediately sized body is interposed than it would if struck directly, and most of all if this [third] is their geometric mean.
4. A wonderful law of nature (which I can verify for spherical bodies, and which seems to be general for all, whether the collision be direct or oblique and whether the bodies be hard or soft) is that the common center of gravity of two, three, or more bodies always moves uniformly in the same direction in the same straight line, before and after their collision.

Let us consider these assertions in reverse order: The fourth states a principle that Newton employed to great effect in his *Principia*. The third allows a strong qualitative test of Huygens's theory. The second announces what came to be known as the conservation of *vis viva*. And the first repudiates Descartes' conservation of motion, but then substitutes for it a principle of conservation of *vectorial* motion—what we now call the conservation of momentum. But to contrast it with Descartes' principle, let me refer to it as "conservation of directional motion."

The full paper in which Huygens derived his results from basic principles was finally published posthumously in 1703. It is a masterpiece.[12] The paper proceeds in two stages, dealing respectively with bodies of the same and differing bulk. The hypotheses from which the derivations proceed are all ones that Huygens was sure the Cartesians held. The key hypothesis in the first half is a principle of relativity:

> The motion of bodies and their equal and unequal speeds are to be understood respectively in relation to other bodies which are considered as at rest, even though perhaps both the former and the latter are involved in another common motion. And accordingly,



**René Descartes (1596–1650).**

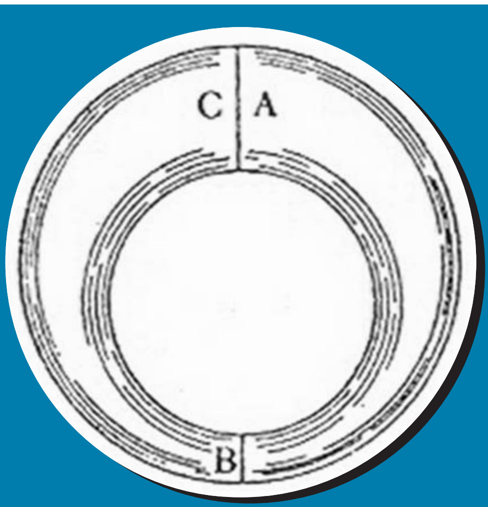



**Figure 1. Benedict de Spinoza's illustration** of Descartes' idea of the conservation of motion. In the circuit with circular boundaries represented here, if the channel is four times wider at *AC* than at *B*, then the velocity of flow around the circuit is four times greater at *B* than at *AC*.

> when two bodies collide with one another, even if both together are further subject to another uniform motion, they will move each other with respect to a body that is carried by the same common motion no differently than if this motion, extraneous to all, were absent.

Huygens used this relativity principle to transform problems into frames of reference in which solutions emerge straightforwardly.

The key hypothesis in the second stage is, "When two hard bodies meet each other, if, after impulse, one of them happens to conserve all the motion that it had, then nothing will be taken from or added to the motion of the other." From this hypothesis Huygens derives the pivotal proposition: "Whenever two bodies collide with one another, the speed of separation is the same, with respect to each other, as that of approach." The argument invokes Torricelli's principle. The rhetorical force of Huygens's paper was to leave those Cartesians who opposed its conclusions grasping at straws.

## A celebrated problem

Huygens's contribution to the conservation of what Leibniz later named *vis viva* did not end with his work on collision. In 1673 he published the *Horologium oscillatorium*.[7] Aside from Newton's *Principia*, the *Horologium* is the most important work in mechanics of the 17th century. Indeed Newton modeled the *Principia* after Huygens's book. Part IV of the *Horologium* solves the celebrated "center of oscillation" problem posed decades earlier by the French cleric Marin Mersenne (1588–1648): What is the length of a simple pendulum with a single bob that beats in unison with a compound pendulum with two or more bobs along its (rigid) string?

Think of a two-bob pendulum. How much does the lower bob slow the motion that the upper one would have in its absence, and vice versa? Huygens's solution proceeded from two hypotheses: Torricelli's principle and the claim that in the absence of resistance "the center of gravity of a rotating pendulum traverses equal arcs in descending and ascending." From these hypotheses he derived two propositions that form the basis of his solution:

1. If any number of bodies all fall or rise, but through unequal distances, the sum of the products of the height of the descent or ascent of each, multiplied by its corresponding magnitude, is equal to the product of the height of the descent or ascent of the center of gravity of all the bodies, multiplied by the sum of their magnitudes.
2. Assume that a pendulum is composed of many weights and, beginning from rest, has completed any part of its whole oscillation. Imagine next that the common bond between the weights has been broken and that each weight converts its acquired velocity upwards and rises as high as it can. Granting all this, the common center of gravity will return to the same height which it had before the oscillation began.

The height in proposition 1 is a surrogate for the square of the acquired velocity. The two propositions together thus impose a relationship between the squares of the velocities acquired by the individual bobs in falling from their separate heights and the square of the velocity acquired by the center of gravity falling from its height. That suffices to solve for the center of oscillation for point-bobs, which Huygens then generalized to the center of oscillation of real bobs of several shapes.

Lagrange attributed the principle of the conservation of *vis viva* to Huygens alone, with no mention of Leibniz, citing Huygens's solution for the center of oscillation rather than his earlier solution of the impact problem.[2] The important point here is that a principle which emerged initially in the context of collisions restricted to hard spheres has turned out, in its more general Galilean form, to make possible the solution of a recalcitrant problem from an entirely different context.

## Leibniz and Newton

Leibniz provoked the *vis viva* controversy in stages, beginning in 1686 and culminating in 1695. In March 1686, two years after he had published his groundbreaking paper on the differential calculus in the new journal *Acta Eruditorum*, Leibniz published a short note in that journal entitled "A Brief Demonstration of a Notable Error of Descartes and Others Concerning a Natural Law."[13] The thrust of the note was to reject the Cartesian equivalence between motive force, which Leibniz agreed is conserved in nature, and quantity of motion, which he argued is not. His argument proceeded from two assumptions: (1) a body falling from a certain height acquires the same force that is necessary to lift it to its original height; and (2) the same force is necessary to raise a body of, say, 1 pound to a height of 4 feet or a body of 4 pounds to a height of 1 foot.

Force, taken as the product of the magnitude of the body and the height from which its velocity can be acquired, is the same for those two bodies. But the velocity acquired by the first body, according to Galileo, is twice the velocity acquired by the second, and hence their quantities of motion are dif-

**Christiaan Huygens (1629–95).**

ferent. From this observation Leibniz concluded that "force is rather to be estimated from the quantity of the effect that it can produce."

Leibniz's 1686 note did not mention *vis viva*, nor did it invoke Huygens's impact results. It did however conclude by proposing that the error to which he was calling attention "is the reason why a number of scholars have recently questioned Huygens's law for the center of oscillation of a pendulum, which is completely true."

Book 1 of Newton's *Principia,* the part that has some bearing on the *vis viva* controversy, went to the printer in April 1686, too soon for him to have seen the *Acta Eruditorum* issue containing Leibniz's note. Newton surely was aware of the controversy by the time of the second (1713) and third (1726) editions of the *Principia*; yet they never mention it. Nevertheless, parts of Book 1 that remained the same in all editions did feed the controversy. For example, the conservation of momentum is presented as a corollary of Newton's laws of motion, with Huygens's center-of-gravity principle, carefully defended, as the next corollary.

In his empirical defense of his laws of motion, Newton indicates how to make corrections for air resistance in measurements of a ballistic pendulum (see figure 2a) to obtain more exacting tests of theories of collision, and then adds that similar corrections can be made for imperfect elasticity of the colliding bodies. Thereby Newton underscores the failure of mass times velocity squared to be conserved when the bodies are not perfectly hard.

Proposition 40 of Newton's *Principia* added another complication. It showed that Galileo's principle of path-independence holds not merely under uniform gravity but for the general case of motion under centripetal forces of any kind. That result has the square of velocity no longer proportional simply to the height of fall but to an integral of the centripetal force over this height. So, without acknowledging as much, proposition 40 supports the importance of velocity squared while making it no longer interchangeable with height of fall.

An important further complication came from Newton's conception of motive force. He expressly tied his centripetal force to Huygens's centrifugal force from the *Horologium*. Huygens had coined *vis centrifuga* to designate the tension in a string holding a body in circular motion. It is a static force, maintaining a state of equilibrium between the string and the revolving object, entirely akin to the tension in a string retaining a vertically suspended weight. Huygens concluded that, in uniform circular motion, the tension is proportional to the weight of the object and, in the infinitesimal limit, to the distance the circle departs from its tangent divided by the square of the time interval of departure.

That's exactly what Newton did with his centripetal force in general curvilinear motion shown in figure 2b, concluding that the force is proportional to the distance $QR$ in the figure divided, in the infinitesimal limit, by the square of the time, as represented by the area $SP \times QT$. The difference is that Newton considered the centripetal force on the object independent of the equal and opposite force on the mechanism producing it.[14] Substituting Newton's mass for Huygens's weight, we see that both men concluded that the force in uniform circular motion is proportional to $mv^2$ divided by the radius. Both men tied the notion of force to static forces in equilibrium, following a usage that had been adopted for some time.

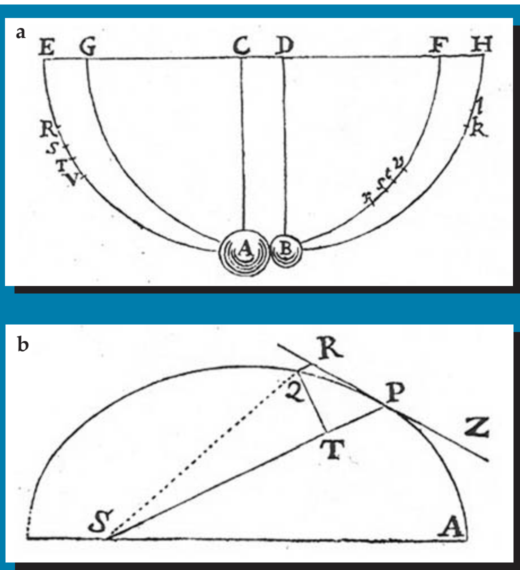



**Figure 2. Diagrams in Isaac Newton's *Principia* (a)** for adding air-resistance corrections in ballistic-pendulum experiments on the impact of hard spheres and **(b)** for determining centripetal force in curvilinear motion.[1]

## Bernoulli's discomfort

Yet another complication came with a 1691 *Acta Eruditorum* article by Jacob Bernoulli (1654–1705) on the compound pendulum's center of oscillation.[15] Expressing discomfort with the "obscure hypothesis" from which Huygens had derived his solution—though not denying it—Bernoulli offered an alternative derivation of the same result by replacing Huygens's hypothesis with the principle of the lever.[16] That principle, unlike Huygens's, is one of static equilibrium, invoking only static forces and what we now call virtual displacements. Bernoulli used it to obtain an equilibrium condition along the pendulum's rigid string that yields the quantities of motion transferred from one bob to another.

The center-of-oscillation problem received continuing attention in the 18th century. As Lagrange pointed out in his opening chapter on dynamics, Newton's three laws, while adequate for the motion of point masses, are not by themselves enough to yield a solution to that problem. The question then, which Bernoulli had initiated, concerned what further principle is to be preferred for solving the compound pendulum problem and a host of related ones. The list of candidates besides Huygens's *vis viva* principle included Jean d'Alembert's generalization of the Bernoulli principle, Leonhard Euler's principle of the moment of momentum, and Pierre de Maupertuis's principle of least action.

## Forces living and dead

Leibniz's 1686 note provoked exchanges with the Cartesians. Descartes' conservation of motion (see figure 1) was difficult to abandon if one believed that all space is filled with matter. The exchanges led Leibniz to refine his position in writings on "dynamics" (the term is his) that were not published until the 19th century.[17] In those writings Leibniz grants the conservation of directional motion, but argues that because it is directional, unlike $mv^2$, it involves reference to other bodies and therefore is not a feature of each body taken unto itself. He concedes that $mv^2$ is not obviously conserved in the collision of soft bodies. But he contends that it is actually conserved via undetected motion of the microphysical parts of the bodies.

Leibniz published one paper on his "new science of dynamics."[18] Entitled "Specimen dynamicum," it appeared in 1695. There he introduced *vis viva* as part of a distinction between living and dead force. His examples of dead force included "centrifugal force and gravitational or centripetal force," along with the forces involved in static equilibrium that, when unbalanced, initiate motion.

"The ancients," he remarks, "so far as is known, had conceived only a science of dead forces, which is commonly referred to as Mechanics, dealing with the lever, the windlass, the inclined plane." Such forces are indeed proportional to the product of bulk and velocity, because "at the very commencement of motion" the space covered varies as the velocity. Living force, which appears in impact, "arises from an infinite number of constantly continued influences of dead forces."

Invoking the metaphysical principle that the effect must equal the cause, Leibniz gave a variant of his 1686 argument: He calculated "the force through the effect produced in using itself up" to conclude that the force transferred from one equal body to another varies as the *square* of the velocity. Leibniz made clear that the metaphysical principle is what establishes the priority of the conservation of living forces in changes of motion.

The *vis viva* controversy that the 18th century inherited from the 17th did indeed concern which quantities are universally conserved: Descartes' motion, Leibniz's *vis viva*, or what we now call momentum. The controversy continued for so long because it involved several further issues. One was the semantic issue of what the term "force" should designate. Less tractable, though not less productive of confusion, was the metaphysical issue Leibniz raised. Then there was the vexing empirical issue of the apparent nonconservation of *vis viva* in the collision of soft bodies.

Much of the study of motion in the 18th century focused on specific problems and on principles from which their mathematical solutions could be derived. The failure of *vis viva* for soft bodies raised concerns about when that principle could safely be applied. And finally, there was the issue raised by Bernoulli: Was it appropriate to take the *vis viva* principle as axiomatic even in cases where it does give the right answer—let alone, as Leibniz urged, taking it to be fundamental to all of mechanics? Little wonder, then, that the controversy lasted so long.

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students who are aiming for a career in science and technology. When I was placed in New York City's public Stuyvesant High School for academically gifted students, I was shocked and awakened by the challenge facing me. Without that challenge, which came mainly from a critical mass of bright, intensely curious students, I would never have become an engineer and a PhD physicist.

I propose that the US government fund and build 435 new free public high schools of science, like my Stuyvesant and the Bronx High School of Science, that would be locally controlled. The schools could be built over seven years at the rate of 63 per year, 1 in each congressional district, plus 6 for the District of Columbia, Guam, and Puerto Rico. High-tech industries would help in many ways.

The cost would be roughly $4 billion per year for seven years. We can afford it. There are bright, creative minds inside youngsters of every skin color, ethnicity, and religion imaginable. Instead of Congress reluctantly granting up to 120 000 special visas each year for talented foreigners to work in our high-tech industries, why not harvest the best minds from among young people born in America? China and India, with a combined population of 2.4 billion, are doing this now.

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## Fixing the record on Descartes' rules for impact

My article "The *Vis Viva* Dispute: A Controversy at the Dawn of Dynamics" (Physics Today, October 2006, page 31) included an unfortunate historical error: René Descartes' rules for impact all first appeared in the 1644 Latin edition of his *Principia* and not in the 1647 French edition, as remarked in the article. The French edition merely expanded his explanation of the rules. Although this error is irrelevant to the article's overall argument, it is nevertheless important to correct, for such careless expositional flourishes have a way of turning into accepted fact when not corrected.

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