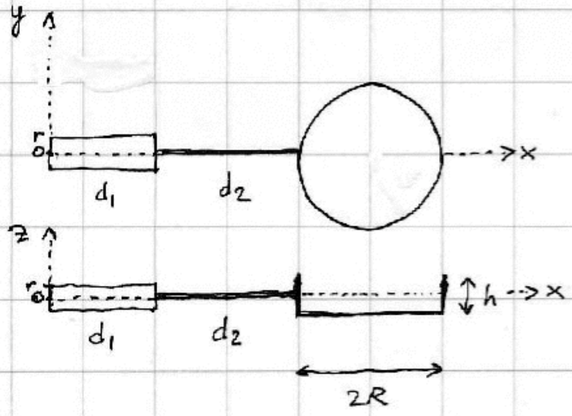
PADELLA



1 = manico cilindrico di legno ρ_L

2 = bacchetta di ferro λ_F

3 = mantello cilindrico della padella σ_A

4 = fondo circolare della padella σ_A

$M_1 = \rho_L \pi r^2 d_1 \qquad \vec{r}_{c_1} = \left(\frac{d_1}{2}, 0, 0\right)$

$M_2 = \lambda_F\, d_2 \qquad \vec{r}_{c_2} = \left(d_1 + \frac{d_2}{2}, 0, \right)$

$M_3 = \sigma_A\, 2\pi R h \qquad \vec{r}_{c_3} = \left(d_1 + d_2 + R, 0, 0\right)$

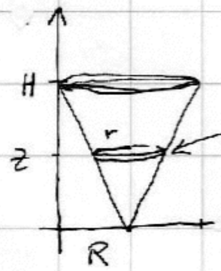
$M_4 = \sigma_A \pi R^2 \qquad \vec{r}_{c_4} = \left(d_1 + d_2 + R, 0, -\frac{h}{2}\right)$

$$\vec{r}_c = \frac{M_1\vec{r}_{c_1} + M_2\vec{r}_{c_2} + M_3\vec{r}_{c_3} + M_4\vec{r}_{c_4}}{M_1 + M_2 + M_3 + M_4} =$$

$$\begin{cases} x_c = \dfrac{M_1\frac{d_1}{2} + M_2\left(d_1 + \frac{d_2}{2}\right) + (M_3 + M_4)(d_1 + d_2 + R)}{M_1 + M_2 + M_3 + M_4} \\ y_c = 0 \\ z_c = \dfrac{-M_4\frac{h}{2}}{M_1 + M_2 + M_3 + M_4} \end{cases}$$

CONO

A) cono omogeneo tutto intero, di altezza H e raggio di base R



$dm = \rho \pi r^2 dz \qquad r = \frac{R}{H} z \qquad dm = \left(\rho\pi\frac{R^2}{H^2}\right) z^2 dz$

$$M = \int dm = \int_0^H \left(\rho\pi\frac{R^2}{H^2}\right) z^2 dz = $$

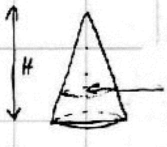
$$= \rho\pi\frac{R^2}{H^2}\cdot\frac{H^3}{3} = \rho\cdot\left(\frac{1}{3}\pi R^2 H\right)$$

$$M z_c = \int z\,dm = \int_0^H \left(\rho\pi\frac{R^2}{H^2}\right) z^3 dz =$$

$$= \rho\pi\frac{R^2}{H^2}\cdot\frac{H^4}{4} = \rho\left(\frac{1}{3}\pi R^2 H\right)\cdot\frac{3}{4}H =$$

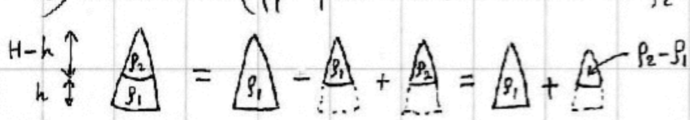
$$= M\cdot\frac{3}{4}H \rightarrow z_c = \frac{3}{4}H.$$

in altre parole, prendendo il cono con la punta in su invece che in giù, si ha



$z_c = \frac{1}{4}H$

B) 2 densità (ρ_1 fino ad altezza h e ρ_2 da h a H):



$$z_c = \frac{M_1\frac{H}{4} + \Delta M\cdot\left[h + (H-h)\frac{1}{4}\right]}{M_1 + \Delta M}.$$

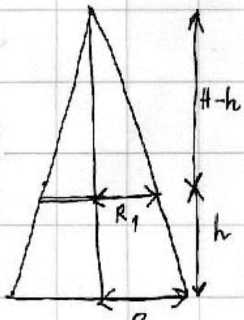
($x_c = y_c = 0$: il c.d.m. è sull'asse del cono)

$M_1 = \rho_1\left(\frac{1}{3}\pi R^2 H\right)$

$\Delta M = (\rho_2 - \rho_1)\cdot\frac{1}{3}\pi R_1^2 (H-h) =$

$= (\rho_2 - \rho_1)\frac{1}{3}\pi R^2\left(\frac{H-h}{H}\right)^2 (H-h) =$

$= (\rho_2 - \rho_1)\frac{1}{3}\pi\left(\frac{R}{H}\right)^2 (H-h)^3.$



$R_1 = R\cdot\left(1 - \frac{h}{H}\right)$